Band Structure in the Excitation Spectrum of Type II Superconductors

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The periodic structure of the mixed state leads to some band splitting in rough analogy to the electrons in a metal. The density of states for tunneling perpendicular to the flux lines shows an appreciable depression at energies $\hbar\omega \approx (\hbar^2/2m)\, k_F |\boldsymbol{l}|$, $\boldsymbol{l}$ being a reciprocal lattice vector of the flux lattice normal to the tunneling surface.

The mixed state of type II superconductors represents a state where the order parameter is not constant in space but varies periodically defining a two dimensional lattice. As is known from the „Geometrical Resonances“[1] a specific scattering of the superconducting electrons takes place at local inhomogeneities of the order parameter. There the question arises whether a periodic arrangement of these inhomogeneities may lead, as in Bragg reflexion, to a structure in the density of states.

In order to find an energy level splitting caused by the periodic array of flux lines in a type II superconductor one has to look for degeneracies in the excitation spectrum of the homogeneous, isotropic superconductor. The single particle excitations with wave vector $\boldsymbol{k}$ of the BCS theory[2] are $E_k = (\varepsilon_k^2 + \Delta_0^2)^{1/2}$, where ε_k is the energy of the free electron and Δ_0 the energy gap. Two states with wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are degenerate if

$$\varepsilon_{k_1} = \varepsilon_{k_2} \tag{1}$$

or

$$\varepsilon_{k_1} = -\varepsilon_{k_2}\,. \tag{2}$$

If both states differ approximately by a lattice vector of the reciprocal lattice corresponding to the mixed state they may be combined in a kind of Bragg reflexion.

As a consequence the degeneration would be removed, i.e. the levels given by Eqs. (1) and (2), respectively, should split. Braggs condition together with either the first or the second energy relation defines two surfaces in the three dimensional k-space where the energy discontinuity will arise. In the nomenclature of ordinary solid state physics these surfaces are the boundaries of the Brillouin zones. The energy bands generated by the discontinuity may overlap along the zone boundaries. That leads to a difference in the band splitting caused by the two energy relations respectively, whereas the magnitude of the gap between the bands should be approximately equal in both cases. In the approximation where the flux lattice is considered as a small perturbation an estimate for that magnitude is that Fourier component of the order parameter belonging to the reciprocal lattice vector involved in the Bragg reflexion. It follows from the geometric shape of the zone boundaries that the overlap is much stronger in the case defined by Eq. (1). There the zone boundaries are the planes one is familiar with by solid state physics. Because of the great absolute value of the involved wave vectors being restricted to the neighborhood of the fermisurface the planes are nearly perpendicular to the surfaces of constant energy E_k and the energy bands would strongly overlap. Thus one would get a vanishing effect in the density of states.

The case defined by Eq. (2) is different. Here the two vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2 = \boldsymbol{k}_1 + \boldsymbol{l}$, $\boldsymbol{l}$ being a reciprocal lattice vector, lie on two spheres, the centers of which are separated from the origin by $+\frac{1}{2}\boldsymbol{l}$ and $-\frac{1}{2}\boldsymbol{l}$ respectively. The spheres cut the surfaces of constant energy in a small angle and the overlap of energy bands should be relatively small. In the following we will be concerned only with that case.

Our task is to investigate by usual Green functions technique whether the periodic inhomogeneities of the mixed state can lead to a splitting of the levels related by Eq. (2).

In the one dimensional case without external field a similar idea has been carried out[3]. — An infinite

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[2] J. BARDEEN, L. N. COOPER, and J. R. SCHRIEFFER, Phys. Rev. **108**, 1175 [1957].

[3] A. P. VAN GELDER, Phys. Rev. **181**, 787 [1969].

system of alternating normal and superconducting layers leads theoretically to a band splitting of the excitation spectrum.

Considering the mixed state it has been shown that the quasiparticle excitations may be taken as Bloch states of the fluxline lattice under the action of a homogeneous magnetic field which represents the external field [4].

Gorkovs Equations

We define the Green functions[5] of the ground-state $|0\rangle$

$$G_\alpha(\boldsymbol{r}, \boldsymbol{r}'; t) = \frac{1}{i}\langle 0|\, T\,\psi_\alpha(\boldsymbol{r}, t)\,\psi_\alpha^+(\boldsymbol{r}', 0)\,|0\rangle\, e^{-iI(\boldsymbol{r}, \boldsymbol{r}')}, \tag{3}$$

$$G_\alpha^+(\boldsymbol{r}, \boldsymbol{r}'; t) = -\frac{1}{i}\langle 0|\, T\psi_\alpha^+(\boldsymbol{r}, t)\,\psi_\alpha(\boldsymbol{r}', 0)\,|0\rangle\, e^{+iI(\boldsymbol{r}, \boldsymbol{r}')}, \tag{4}$$

$$F(\boldsymbol{r}, \boldsymbol{r}'; t) = \frac{1}{i}\langle 0|\, T\,\psi_\downarrow(\boldsymbol{r}, t)\,\psi_\uparrow(\boldsymbol{r}', 0)\,|0\rangle\, e^{+iI(\boldsymbol{r}, \boldsymbol{r}')}, \tag{5}$$

$$F^+(\boldsymbol{r}, \boldsymbol{r}'; t) = -\frac{1}{i}\langle 0|\, T\,\psi_\uparrow^+(\boldsymbol{r}, t)\,\psi_\downarrow^+(\boldsymbol{r}', 0)\,|0\rangle\, e^{-iI(\boldsymbol{r}, \boldsymbol{r}')} \tag{6}$$

with

$$I(\boldsymbol{r}, \boldsymbol{r}') = \frac{e}{\hbar c}\int_0^1 [(\boldsymbol{r} - \boldsymbol{r}')\,\boldsymbol{A}(\boldsymbol{r}' + (\boldsymbol{r} - \boldsymbol{r}')\,\tau)]\,\mathrm{d}\tau \tag{7}$$

and denote by a bar the corresponding matrix Green function

$$\bar{G}(\boldsymbol{r}, \boldsymbol{r}'; t) = \begin{pmatrix} G(\boldsymbol{r}, \boldsymbol{r}'; t) & F(\boldsymbol{r}, \boldsymbol{r}'; t) \\ F^+(\boldsymbol{r}, \boldsymbol{r}'; t) & G^+(\boldsymbol{r}, \boldsymbol{r}'; t) \end{pmatrix}. \tag{8}$$

Gorkovs equations[6] may be written in matrix form as

$$\begin{pmatrix} \hbar\omega + \mu - \frac{1}{2m}\left(\frac{\hbar}{i}\nabla_r + \hbar\boldsymbol{k} - \frac{e}{c}\boldsymbol{A} \pm \left(\boldsymbol{M} + \frac{e}{c}\boldsymbol{A}\right)\right)^2, & \Delta(\boldsymbol{r}) \\ -\Delta^*(\boldsymbol{r}), & -\hbar\omega + \mu - \frac{1}{2m}\left(\frac{\hbar}{i}\nabla_r + \hbar k + \frac{e}{c}\boldsymbol{A} \pm \left(\boldsymbol{M} + \frac{e}{c}\boldsymbol{A}\right)\right)^2 \end{pmatrix} \bar{G}_\omega(\boldsymbol{k}, \boldsymbol{r}) = \hbar\,\bar{1} \tag{9}$$

where we have used the Fourier-Transformation for the time variable t and the coordinate difference $(\boldsymbol{r} - \boldsymbol{r}')$

$$\bar{G}(\boldsymbol{r}, \boldsymbol{r}'; t) = \frac{1}{(2\pi)^4}\int_{-\infty}^{+\infty}\exp\{-i\omega t + i\boldsymbol{k}(\boldsymbol{r} - \boldsymbol{r}')\} \cdot \bar{G}_\omega(\boldsymbol{k}, \boldsymbol{r})\,\mathrm{d}\omega\,\mathrm{d}^3\boldsymbol{k} \tag{10}$$

and the abbreviation

$$\boldsymbol{M} = \frac{e}{c}\sum_{n=0}^{\infty}\frac{(-i)^n}{(n+2)!}(\nabla_k\nabla_r)^n\,[i\nabla_k \times \boldsymbol{H}(\boldsymbol{r})]\,. \tag{11}$$

The upper (lower) sign in Eq. (9) is chosen if it is operated on the first (second) column, respectively, of the Green function matrix. $\boldsymbol{A}(\boldsymbol{r})$ ($\mathrm{rot}\,\boldsymbol{A} = \boldsymbol{H}$) is the vector potential in arbitrary gauge and $\Delta(\boldsymbol{r})$ is the order parameter. Both quantities are determined by those equations which keep the procedure self-consistent.

$$\Delta^*(\boldsymbol{r}) = -\frac{iV}{(2\pi)^4}\int_{-\infty}^{+\infty}F_\omega^+(\boldsymbol{k}, \boldsymbol{r})\,\mathrm{d}\omega\,\mathrm{d}^3\boldsymbol{k}\,, \tag{12}$$

$$\mathrm{rot}\,\mathrm{rot}\,\boldsymbol{A}(\boldsymbol{r}) = -\frac{4\pi e\hbar}{(2\pi)^4 mc}\cdot\int_{-\infty}^{+\infty}e^{i\omega\varepsilon}(\nabla_r + 2i\boldsymbol{k})\,G_\omega(\boldsymbol{k}, \boldsymbol{r})\,\mathrm{d}\omega\,\mathrm{d}^3\boldsymbol{k}\,. \tag{13}$$

Next we wish to exploit the periodicity for the two dimensional flux lattice. If $\boldsymbol{g}$ means some lattice vector, we have the condition

$$\boldsymbol{A}(\boldsymbol{r} + \boldsymbol{g}) = \boldsymbol{A}(\boldsymbol{r}) + \nabla\varphi_g(\boldsymbol{r}) \tag{14}$$

which may be regarded as a gauge transformation and thus

$$\Delta^*(\boldsymbol{r} + \boldsymbol{g}) = \Delta^*(\boldsymbol{r})\exp\{-(2ie/\hbar c)\,\varphi_g(\boldsymbol{r})\}\,. \tag{15}$$

$G_\omega(\boldsymbol{k}, \boldsymbol{r})$, $G_\omega^+(\boldsymbol{k}, \boldsymbol{r})$, $\boldsymbol{H}(\boldsymbol{r})$, $|\Delta(\boldsymbol{r})|$ are periodic, $F_\omega^+(\boldsymbol{k}, \boldsymbol{r})$ transforms like $\Delta^*(\boldsymbol{r})$ as indicated in Eq. (15) and $F_\omega(\boldsymbol{k}, \boldsymbol{r})$ like the complex conjugate. φ_g depends on the special choice of the gauge, however it is restricted by the fact that the total flux Φ through the unit cell is given and related to φ_g by

$$\Phi \equiv \int_{u.c.} H_z(\boldsymbol{r})\,\mathrm{d}f = \varphi_{g_1}(\boldsymbol{g}_2) - \varphi_{g_1}(\boldsymbol{o}) - \varphi_{g_2}(\boldsymbol{g}_1) + \varphi_{g_2}(\boldsymbol{o}) \tag{16}$$

where we took the magnetic field in the direction of the z-axis. $\boldsymbol{g}_1$, $\boldsymbol{g}_2$ are the basis vectors of the lattice. Thus if the flux does not vanish, $\boldsymbol{A}(\boldsymbol{r})$ cannot be purely periodic whatever a gauge is chosen. On the other hand we want to expand the Greens functions and the corresponding Eqs. (9), (12) and

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(13) in Fourier series with respect to the coordinate $\boldsymbol{r}$. To this end we remove the phase factors in the following quantities

$$\Delta^{(*)}(\boldsymbol{r}) = |\Delta(\boldsymbol{r})| \exp\{\pm (2ie/\hbar c)\,\chi(\boldsymbol{r})\},$$
$$F_{\omega}^{(+)}(\boldsymbol{k}, \boldsymbol{r}) = \tilde{F}_{\omega}^{(+)}(\boldsymbol{k}, \boldsymbol{r}) \exp\{\pm (2ie/\hbar c)\,\chi(\boldsymbol{r})\}$$

(the upper sign refers to the starred quantities) by adding the phase to the vector potential

$$\boldsymbol{A}(\boldsymbol{r}) \to \boldsymbol{A}(\boldsymbol{r}) + \nabla\chi(\boldsymbol{r}) \equiv \tilde{\boldsymbol{A}}(\boldsymbol{r}) \tag{17}$$

which differs from a gauge transformation by the fact that the second partial derivatives of $\chi(\boldsymbol{r})$ may not exist at some singular point.

In the ABRIKOSOV model[7] of type II superconductivity that point (x_0, y_0) represents the center of a flux line, where the order parameter becomes zero and behaves like $\Delta(x, y)$ $\alpha(\zeta - \zeta_0)$, ζ being the complex variable $x + iy$. $\chi(\boldsymbol{r})$ is a linear function of the argument of $(\zeta - \zeta_0)$ hence indeterminate at ζ_0. The second partial derivatives become infinite at ζ_0.

The functions $|\Delta(\boldsymbol{r})|$ and $\tilde{\boldsymbol{A}}(\boldsymbol{r})$ are periodic and may be expanded in a two dimensional Fourier series, if we exclude the singular point by a suitable redefinition of the functions in the immediate neighborhood of that point. We shall not go further in mathematical details. The only question is whether this procedure remains selfconsistent because $|\Delta(\boldsymbol{r})|$ and $\tilde{\boldsymbol{A}}(\boldsymbol{r})$ are inversely determined by the solution of Gorkovs equations through Eq. (12) and (13). We are interested only in periodic solutions of Eq. (9) and may thus expand the Green functions in a Fourier series, too. The magnetic field we assume to be in the local domain so that we may set $\boldsymbol{M} = 0$, Eq. (11). All functions are independent of the z-component of $\boldsymbol{r}$ and we define

$$\tilde{\boldsymbol{A}}(\boldsymbol{r}) = \sum_{l} \boldsymbol{a}_l \exp\{i(\boldsymbol{l}\boldsymbol{r})\}, \tag{18a}$$

$$|\Delta(\boldsymbol{r})| = \sum_{l} \Delta_l \exp\{i(\boldsymbol{l}\boldsymbol{r})\}, \tag{18b}$$

$$\bar{G}_{\omega}(\boldsymbol{k}, \boldsymbol{r}) = \sum_{l} \bar{G}_l \exp\{i(\boldsymbol{l}\boldsymbol{r})\} \tag{18c}$$

where $\boldsymbol{l}$ is a vector of the reciprocal lattice the z-component being zero. We write Eq. (9) as

$$\left(\hbar\omega + \mu - \frac{\hbar^2}{2m}(\boldsymbol{k} + \boldsymbol{l})^2\right) G_l + \sum_{l'} \Delta_{l-l'} F_{l'}^{+} = \hbar\,\delta_{l,0}\,, \tag{19a}$$

$$-\sum_{l'} \Delta_{l-l'} G_{l'} + \left(-\hbar\omega + \mu - \frac{\hbar^2}{2m}(\boldsymbol{k} + \boldsymbol{l})^2\right) F_l^{+} - \sum_{l'} \frac{e\hbar}{mc}(\boldsymbol{a}_{l-l'}(\boldsymbol{l} + \boldsymbol{l}' + 2\boldsymbol{k}))\, F_{l'}^{+} - \sum_{l',l''} \frac{e^2}{mc^2}(\boldsymbol{a}_{l-l'-l''}\boldsymbol{a}_{l''})\, F_{l'}^{+} = 0\,. \tag{19b}$$

The remaining two equations need not to be considered. Eqs. (12) and (13) read now

$$\Delta_l = -\frac{iV}{(2\pi)^4} \int_{-\infty}^{+\infty} F_l^{+}\, \mathrm{d}\omega\, \mathrm{d}^3\boldsymbol{k}\,, \tag{20}$$

$$[\boldsymbol{l} \times [\boldsymbol{l} \times \boldsymbol{a}_l]] = \frac{4\pi e\hbar i}{(2\pi)^4 mc} \int_{-\infty}^{+\infty} e^{i\omega s}(2\boldsymbol{k} + \boldsymbol{l})\, G_l\, \mathrm{d}\omega\, \mathrm{d}^3\boldsymbol{k}\,. \tag{21}$$

In the next part we shall try an approach to the solution of Eq. (19) by means of an approximation being the analogue of the nearly free electron model of a metal.

Nearly Free Quasiparticles

We assume that in the Fourier expansion Eqs. (18a, b, c) only the terms of small wave vector are important. Furthermore we postulate that the energy belonging to those terms in Eq. (19) is small compared with the unperturbed excitation energy of a quasiparticle with wave vector $\boldsymbol{k}$. The last assumption is realized if we restrict the wave vector in Eq. (19) to values separated well enough from the fermisurface. The first condition seems artificial and may be reasonable only if the spatial variation of order parameter and magnetic field are small. That condition is fulfilled in the mixed state if the temperature lies near T_c, however we are dealing with zero temperature. Therefore, and it is not the aim of this paper, we cannot give a full description of the mixed state. We are mainly concerned with the effect coming from the symmetry of an ordered lattice of flux lines disregarding the structure of a single vortex.

If we take $|\Delta(\boldsymbol{r})|$ and $\tilde{\boldsymbol{A}}(\boldsymbol{r})$ as known, e.g. through a solution of the Ginsburg-Landau equations, Eq. (19) represents a system of linear equations for the transformed Green functions. To find their poles we have to search for the zeros of the appropriate determinant. Under the above conditions the system would reduce for a quadratic lattice (4 nearest neighbors) to 10 and for a hexagonal lattice (6 near-

[7] A. A. ABRIKOSOV, Sov. Phys. JETP **5**, 1174 [1957].

est neighbors) to 14 linear equations for the unknown quantities G_l, F_l^+. We need not solve that determinant for the zeros, as we are only interested in the significant change of the spectrum which occurs at values $\boldsymbol{k}_0$ defined by

$$-\frac{\hbar^2}{2m}(\boldsymbol{k}_0+\boldsymbol{l})^2+\mu=\frac{\hbar^2}{2m}\boldsymbol{k}_0^2-\mu \qquad (22)$$

because there we expect the degenerate level to split. If $\boldsymbol{k}$ lies near these values for some fixed, not necessary nearest neighbor vector $\boldsymbol{l}$, the system reduces to 4 equations in which only the Green functions combined by $\boldsymbol{l}$ enter, namely G_0, G_1, F_0^+, F_1^+. The indices $_0$ and $_1$ stand for the vector $\boldsymbol{o}$ and $\boldsymbol{l}$, respectively.

$$\left(\hbar\omega+\mu-\frac{\hbar^2}{2m}\boldsymbol{k}^2\right)G_0+\Delta_0F_0^+-\Delta_1F_1^+=\hbar, \qquad (23\,\mathrm{a})$$

$$\left(\hbar\omega+\mu-\frac{\hbar^2}{2m}(\boldsymbol{k}+\boldsymbol{l})^2\right)G_1+\Delta_1F_0^+-\Delta_0F_1^+=0, \qquad (23\,\mathrm{b})$$

$$\left(-\hbar\omega+\mu-\frac{\hbar^2}{2m}\boldsymbol{k}^2\right)F_0^++\frac{e\hbar}{mc}(\boldsymbol{a}_l(2\boldsymbol{k}+\boldsymbol{l}))F_1^+-\frac{e^2}{mc^2}\sum_l|\boldsymbol{a}_l|^2F_0^+-\Delta_0G_0-\Delta_1G_1=0, \qquad (23\,\mathrm{c})$$

$$\left(-\hbar\omega+\mu-\frac{\hbar^2}{2m}(\boldsymbol{k}+\boldsymbol{l})^2\right)F_1^+-\frac{e\hbar}{mc}(\boldsymbol{a}_l(2\boldsymbol{k}+\boldsymbol{l}))F_0^+-\frac{e^2}{mc^2}\sum_l|\boldsymbol{a}_l|^2F_1^+-\Delta_1G_0-\Delta_0G_1=0. \qquad (23\,\mathrm{d})$$

We have used the symmetries $\Delta_l=\Delta_{-l}=\Delta_l^*$, $\boldsymbol{a}_l=-\boldsymbol{a}_{-l}=-\boldsymbol{a}_l^*$. With

$$\beta=\frac{e^2}{2mc^2}\sum_l|\boldsymbol{a}_l|^2,\qquad \alpha=\frac{ie\hbar}{mc}(\boldsymbol{a}_l(2\boldsymbol{k}+\boldsymbol{l})),$$

$$\tilde{\varepsilon}_0=\frac{\hbar^2}{2m}\boldsymbol{k}^2-\mu+\beta,\qquad \hbar\tilde{\omega}=\hbar\omega+\beta,$$

$$\tilde{\varepsilon}_1=\frac{\hbar^2}{2m}(\boldsymbol{k}+\boldsymbol{l})^2-\mu+\beta,$$

we get for the determinant of the system (23)

$$D=((\hbar\tilde{\omega})^2-\tilde{\varepsilon}_0^2-\Delta_0^2-\Delta_1^2)((\hbar\tilde{\omega})^2-\tilde{\varepsilon}_1^2-\Delta_0^2-\Delta_1^2)-\Delta_1^2(\tilde{\varepsilon}_0-\tilde{\varepsilon}_1)^2-4\Delta_0^2\Delta_1^2-\alpha^2(\hbar\tilde{\omega}-\tilde{\varepsilon}_0)(\hbar\tilde{\omega}-\tilde{\varepsilon}_1). \qquad (24)$$

In the case of vanishing field D becomes

$$D=((\hbar\omega)^2-\varepsilon_0^2-\Delta_0^2)((\hbar\omega)^2-\varepsilon_1^2-\Delta_0^2)$$

with zeros at

$$(\hbar\omega)_{1,2,3,4}=\begin{cases}\pm\sqrt{\varepsilon_0^2+\Delta_0^2}\\ \pm\sqrt{\varepsilon_1^2+\Delta_0^2}\end{cases}$$

which are the BCS excitation energies. If we take the vectorpotential as small perturbation, we see immediately from Eq. (24) that for $\varepsilon_0=-\varepsilon_1$ there are no states with energy $(\hbar\omega)_l$. Forbidden energy zones appear and under the assumption of a small perturbation the level splitting at $\varepsilon_0=-\varepsilon_1$ is given by

$$(\hbar\omega)^2=\varepsilon_0^2+\Delta_0^2\pm\sqrt{4\Delta_1^2(\varepsilon_0^2+\Delta_0^2)+\alpha^2\Delta_0^2}. \qquad (25)$$

The geometric shape of that surface in momentum space where the splitting arises is easily determined from condition (22).

$$\varepsilon_{k+\frac{1}{2}l}=-(\hbar^2/8m)\boldsymbol{l}^2. \qquad (26)$$

$\boldsymbol{k}$ lies on a sphere with radius smaller than the Fermi wave vector k_F by an amount $(1/8k_\mathrm{F})\boldsymbol{l}^2$ and the center being separated from the origin by $-\frac{1}{2}\boldsymbol{l}$. It is interesting to note that the surface definition depends on the z-coordinate, too, contrary to the case of usual Bragg reflexion $\varepsilon_0=+\varepsilon_1$, where the definition of the boundaries of Brillouin zones in a two dimensional lattice is independent of the third axis.

As the spheres of constant energy deviate only a little from the sphere (26) the new excitation energy does not vary much along the latter surface. We get an estimate for the band overlap by the quantity

$$\left|\frac{\partial\hbar\omega}{\partial\boldsymbol{k}}\right|\approx\frac{\hbar^2}{m}|\boldsymbol{l}| \qquad (27)$$

in contrast to the corresponding expression in Bragg reflexion

$$\left|\frac{\partial\hbar\omega}{\partial\boldsymbol{k}}\right|\approx\frac{\hbar^2}{m}|\boldsymbol{k}| \qquad (28)$$

which is greater by a factor 10^3—10^4. It is this reason why one will not find a structure in the density of states resulting from Bragg reflexion. If we take $\varepsilon_0\gg\Delta_0$ the separation of the bands is approximately

$$\delta(\hbar\omega)\approx\Delta_1. \qquad (29)$$

Tunneling Density of States

We shall restrict the calculation to energies $\hbar\omega$ greater than zero measured relative to the Fermi energy. We suppose the tunneling surface to extend in y- and z-direction. The coordinates are chosen so that the x-axis coincides with the direction of $\boldsymbol{l}$. The wave vector components k_y, k_z of the tunneling electrons can be assumed to be nearly zero. We are interested in a tunneling density of states $N_\omega(k_y, k_z)$

which gives the probability that an incident electron with k_y, k_z fixed may be added at x. That density is defined by

$$N_\omega(k_y, k_z) \alpha \sum_n \delta(\hbar\omega - \varepsilon_n) \left|\langle 0| \psi_{ky,kz}(x) |n\rangle\right|^2 \quad (30)$$

where n denotes a quantum state of the whole system and ε_n the appropriate energy relative to the Fermi energy. Eq. (30) leads to

$$N_\omega(k_y, k_z) \alpha - \frac{1}{2\pi^2\hbar} \int_{-\infty}^{+\infty} dk_x \frac{1}{L^2} \int_0^L dy\, dz\, \mathrm{Im}\, G_\omega(\boldsymbol{k}, \boldsymbol{r}) \quad (31)$$

where we used the definition of the Green function, Eq. (3). Next we have to evaluate $G_\omega(\boldsymbol{k}, \boldsymbol{r})$ from the system of Eq. (23). We get

$$G_0 = \hbar D^{-1}[(\hbar\tilde{\omega} + \tilde{\varepsilon}_0)((\hbar\tilde{\omega})^2 - \tilde{\varepsilon}_1^2 - \Delta_0^2 - \Delta_1^2) - \alpha^2(\hbar\tilde{\omega} - \tilde{\varepsilon}_1) + \Delta_1^2(\tilde{\varepsilon}_0 - \tilde{\varepsilon}_1)] \quad (32)$$

and

$$G_1 = \hbar D^{-1}[\Delta_0\Delta_1(2\hbar\tilde{\omega} + \tilde{\varepsilon}_0 + \tilde{\varepsilon}_1) + i\alpha(\Delta_0^2 - \Delta_1^2)]. \quad (33)$$

It is well set that one can calculate macroscopic quantities as the free energy and by this way the vector potential and the order parameter in the framework of a local approximation, because finally the Green functions are always integrated over the energy and momentum variables. But if one is interested in the poles of the Green function this procedure clearly is not applicable, because the terms of an expansion in powers of a nonlocal quantity contain these poles with increasing order. In this sense our approximation procedure differs, i.e. is not so restrictive, from the usual local approximation. On the other hand the Green functions resulting from Eqs. (32) and (33) are not at all sufficient to describe the macroscopic mixed state.

Through G_1, Eq. (33) and (18c), the tunneling density of states will exhibit a term oscillating with the coordinate x. Furthermore if $\hbar\omega$ is far apart of a zero $\hbar\omega_i(k_x)$ of D where k_x satisfies $\varepsilon_k = -\varepsilon_{k+l}$ with k_y, k_z fixed, then we get for the tunneling density of states dropping the correction term G_1

$$N_\omega(k_y, k_z) \alpha ((\hbar\omega)^2 - \Delta_0^2)^{-1/2} \times \left[\left(\hbar\omega\, 1 + \frac{k_y^2 + k_z^2}{2k_F^2}\right) - \frac{(\hbar\omega)^2 - \Delta_0^2}{2\mu}\right]. \quad (34)$$

The magnetic field has been regarded as a small perturbation.

The case $\hbar\omega \approx \hbar\omega_i(k_x)$ will lead to some change in the density given by Eq. (34). As $\hbar\omega > 0$, there are two zeros of D and we will be interested in values $\hbar\omega$ lying in the interval between these zeros where the density is expected to decrease.

The evaluation of the integral over k_x in Eq. (31) is done if one knows the poles of G_ω, i.e. the zeros of D regarding the variable k_x. Generally there are eight zeros, four emerging from a translation $k_x^{5,\ldots,8} = k_x^{1,\ldots,4} + l_x$ of the others. $k_x^5, \ldots, k_x^8$ are of no importance in the case which leads to Eq. (34), because their residues are small. However in the other case $\hbar\omega$ lies in the interval defined by Eq. (25), and four of the eight zeros take on complex values. The remaining four real zeros are characterized by $\varepsilon_k \cdot \varepsilon_{k+l} > 0$. For two of these zeros the residue is, as in the former case, vanishing small. One is left with two zeros which lead in the density of states to one half the right side of Eq. (34).

Thus $N_\omega(k_y, k_z)$ is reduced by a factor $\frac{1}{2}$ in the energy intervals where scattering between the states ε_k and $-\varepsilon_{k+l}$ removes the corresponding degenerate level E_k. The width of an interval, Δ_1 [Eq. (29)], decreases with increasing length of the reciprocal lattice vector $\boldsymbol{l}$ involved in the scattering. For $|\boldsymbol{l}|$ smallest, the order of magnitude of Δ_1 is Δ_0 and the distance between two succeeding intervals (i.e. the band width) is of the same order. However, there we have $\hbar\omega$ near the superconducting energy gap Δ_0, and the calculation should not be taken too seriously. In view of our approximations the result applies to the higher excitations, where it estimates the correction to the BCS excitation energies due to the scattering by the flux line lattice. The variation in the tunneling density of states should be apparent in an appropriate tunneling experiment with very clean type II superconductors.